

# QED corrections to DIS with a tagged photon at HERA

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**Abstract:** We report on the calculation of the QED radiative corrections to deep inelastic scattering with a tagged photon with next-to-leading logarithmic accuracy. Numerical results are given for different experimental setups for the case of the HERA collider.

## 1 Introduction

One of the major tasks of the experiments at the HERA  $ep$  collider is the determination of the structure functions of the proton,  $F_2(x, Q^2)$  and  $F_L(x, Q^2)$ , over a broad range of the kinematical variables. Especially the extension of the measurements from fixed target experiments to the range of small Bjorken  $x < 10^{-4}$  and for  $Q^2$  of a few GeV $^2$  is of particular interest, as it provides a challenge for theoretical attempts to understand the details of the dynamics of quarks and gluons inside the nucleon.

There exist several ways to separately extract  $F_L(x, Q^2)$  and  $F_2(x, Q^2)$  from the experimental data. Besides indirect methods that rely on extrapolations or QCD fits (see e.g., [1]), there exist two direct ones. The first requires to run the collider at different center-of-mass energies. The second method was suggested by Krasny et al. [2] and utilizes radiative events with an exclusive hard photon registered in the forward photon detector (PD). Such a device is actually part of the luminosity monitoring system of the H1 and ZEUS experiments. The idea of this method is that emission of photons in a direction close to the incoming electron corresponds to a reduced effective beam energy. This effective beam energy for each radiative event is determined from the energy of the hard photon observed (tagged) in the PD.

The potential of this method is supported by recent preliminary results from the H1 collaboration of an analysis for  $F_2$  [3] (for earlier analyses that did not take into account QED radiative corrections see [4, 5]). The feasibility of a determination of  $F_L$  was studied in [6].

It is the purpose of this contribution to review recent calculations of the QED radiative corrections [7–10] to the process under consideration.

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## 2 Kinematics and Lowest Order Cross Section

A convenient set of kinematic invariants that takes into account the energy loss from the collinearly radiated photon in the process

$$e(p) + p(P) \rightarrow e(p') + \gamma(k) + X(P'), \quad (1)$$

is given by [2]:

$$\hat{Q}^2 = -(p - p' - k)^2, \quad \hat{x} = \frac{\hat{Q}^2}{2P \cdot (p - p' - k)}, \quad \hat{y} = \frac{P \cdot (p - p' - k)}{P \cdot (p - k)}. \quad (2)$$

We restrict ourselves to the case where the polar angle  $\vartheta_\gamma$  of the photon (measured with respect to the incident electron beam) is assumed to be very small,  $\vartheta_\gamma \leq \vartheta_0$ , with  $\vartheta_0$  being about 0.45 mrad in the case of the PD of H1. Therefore, it is emitted almost collinearly. In addition we require that the angle  $\theta$  of the final electron be large compared to the angle  $\vartheta_0$ .

It is convenient to denote by  $z$  the energy fraction of the electron after initial state radiation of a tagged collinear photon,

$$z = \frac{2P \cdot (p - k)}{S} = \frac{E_e - E_\gamma}{E_e} = \frac{\hat{Q}^2}{\hat{x}\hat{y}S}, \quad \text{with } S = 2P \cdot p, \quad (3)$$

where  $E_e$  is the electron beam energy, and  $E_\gamma$  represents the energy deposited in the forward PD.

The Born cross section, integrated over the solid angle of the photon detector ( $0 \leq \vartheta_\gamma \leq \vartheta_0$ ,  $\vartheta_0 \ll \theta$ ) takes a factorized form (see also [2, 7–9, 11]):

$$\frac{1}{\hat{y}} \frac{d^3\sigma_{\text{Born}}}{d\hat{x} d\hat{y} dz} = \frac{\alpha}{2\pi} P(z, L_0) \tilde{\Sigma}(\hat{x}, \hat{y}, \hat{Q}^2), \quad (4)$$

where

$$\tilde{\Sigma}(\hat{x}, \hat{y}, \hat{Q}^2) = \frac{2\pi\alpha^2(-\hat{Q}^2)}{\hat{Q}^2\hat{x}\hat{y}^2} \left[ 2(1 - \hat{y}) - 2\hat{x}^2\hat{y}^2 \frac{M^2}{\hat{Q}^2} + \left( 1 + 4\hat{x}^2 \frac{M^2}{\hat{Q}^2} \right) \frac{\hat{y}^2}{1 + R} \right] F_2(\hat{x}, \hat{Q}^2),$$

with

$$\begin{aligned} P(z, L_0) &= \frac{1+z^2}{1-z} L_0 - \frac{2z}{1-z}, \quad L_0 = \ln \left( \frac{E_e^2 \vartheta_0^2}{m^2} \right), \quad \alpha(-\hat{Q}^2) = \frac{\alpha}{1 - \Pi(-\hat{Q}^2)}, \\ R &= R(\hat{x}, \hat{Q}^2) = \left( 1 + 4\hat{x}^2 \frac{M^2}{\hat{Q}^2} \right) \frac{F_2(\hat{x}, \hat{Q}^2)}{2\hat{x}F_1(\hat{x}, \hat{Q}^2)} - 1. \end{aligned} \quad (5)$$

The quantities  $F_2$  and  $F_1$  denote the proton structure functions,  $M$  and  $m$  are the proton and electron masses. We explicitly include the correction from vacuum polarization  $\Pi(-\hat{Q}^2)$  in the virtual photon propagator, and we neglect the contributions from Z-boson exchange and  $\gamma$ -Z interference, because we are interested mostly in the kinematic region of small momentum transfer  $\hat{Q}^2$ .

### 3 Radiative Corrections

The cross section (4) describes the process (1) to lowest order in perturbation theory. We shall restrict our discussion to the model-independent corrections to the electron line. The radiative corrections to this cross section are composed of contributions by corrections due to virtual photon exchange, soft photon emission, and emission of a second hard photon, with one (or both) of the hard photons being tagged in the PD. Because of its coarse granularity, we shall assume that the PD cannot measure photons individually but only their total energy when two hard photons simultaneously hit the PD in different locations.

As is well known (see e.g., the discussion in [12]), radiation of an (additional) hard photon influences the experimental determination of the kinematical variables. Therefore, the calculation of the contributions from the emission of two hard photons will depend on the chosen method. On the other hand, the virtual corrections and the contributions from soft photon emission in addition to the hard one will be independent of this choice.

#### 3.1 Virtual and Soft Corrections

The virtual and soft corrections to the lowest order cross section were obtained in [8, 9]. Since the tagged photon is emitted almost collinearly, the cross section takes again a simple, factorized form,

$$\frac{1}{\hat{y}} \frac{d^3\sigma_{V+S}}{d\hat{x} d\hat{y} dz} = \frac{\alpha^2}{4\pi^2} [P(z, L_0)\tilde{\rho} - T] \tilde{\Sigma}(\hat{x}, \hat{y}, \hat{Q}^2), \quad (6)$$

with

$$\begin{aligned} \tilde{\rho} &= 2(L_Q - 1) \ln \frac{\Delta^2}{Y} + 3L_Q + 3 \ln z - \ln^2 Y - \frac{\pi^2}{3} - \frac{9}{2} + 2\text{Li}_2\left(\frac{1+c}{2}\right), \\ T &= \frac{1+z^2}{1-z}(A \ln z + B) - \frac{4z}{1-z}L_Q \ln z - \frac{2-(1-z)^2}{2(1-z)}L_0 + \mathcal{O}(\text{const}), \\ A &= -L_0^2 + 2L_0L_Q - 2L_0 \ln(1-z), \quad B = [\ln^2 z - 2\text{Li}_2(1-z)]L_0, \\ L_Q &= \ln \frac{\hat{Q}^2}{zm^2}, \quad \text{Li}_2(t) = - \int_0^t \frac{du}{u} \ln(1-u). \end{aligned} \quad (7)$$

Here  $\Delta$  denotes the infrared cutoff for the emission of a soft photon in addition to the hard one, i.e.,  $E_{\text{soft}} \leq \Delta E_e$ ,

$$Y \equiv \frac{E'_e}{E_e} = z(1-\hat{y}) + \hat{x}\hat{y} \frac{E_p}{E_e}, \quad \text{and} \quad c \equiv \cos\theta = \frac{z(1-\hat{y})E_e - \hat{x}\hat{y}E_p}{z(1-\hat{y})E_e + \hat{x}\hat{y}E_p}. \quad (8)$$

It should be noted that in (6,7) and also in the contributions from double hard bremsstrahlung we retain only terms with double or single large logarithms of the small electron mass  $m$ , i.e., terms of order  $\alpha^2 L^2$  and  $\alpha^2 L$ , with  $L$  being one of  $L_0$  or  $L_Q$ . As the lowest order cross section (4) is of order  $\alpha L$  relative to the DIS cross section, we denote the terms of order  $\alpha^2 L^2$  as leading (LL) and those of order  $\alpha^2 L$  as next-to-leading logarithmic (NLL) ones.

### 3.2 Double Hard Bremsstrahlung

Besides the soft and virtual corrections to the lowest order process, we have to consider also the corrections from hard bremsstrahlung, which in the present case corresponds to double hard bremsstrahlung.

In the calculation of the contributions from the emission of two hard photons, it is convenient to decompose the phase space into three regions [9]: *i*) both hard photons hit the forward photon detector, i.e., both are emitted within a narrow cone around the electron beam ( $\vartheta_{1,2} \leq \vartheta_0$ ,  $\vartheta_0 \ll 1$ ); *ii*) one photon is tagged in the PD, while the other is collinear to the outgoing electron ( $\vartheta'_2 \equiv \angle(\vec{k}_2, \vec{p}') \leq \vartheta'_0$ ); and finally *iii*) the second photon is emitted at large angles (i.e., outside the defined narrow cones) with respect to both incoming and outgoing electron momenta. The last kinematic domain is denoted as the semi-collinear one. For the sake of simplicity, we shall always assume that  $m/E_e \ll \vartheta'_0 \ll 1$ .

The contribution from the kinematic region *i*), with both hard photons being tagged, and only the sum of their energies measured, does not depend on the determination of the kinematic variables and reads

$$\begin{aligned} \frac{1}{\hat{y}} \frac{d^3\sigma_i^{\gamma\gamma}}{d\hat{x} d\hat{y} dz} = & \frac{\alpha^2}{8\pi^2} L_0 \left[ L_0 \left( P_\Theta^{(2)}(z) + 2 \frac{1+z^2}{1-z} \left( \ln z - \frac{3}{2} - 2 \ln \Delta \right) \right) \right. \\ & + 6(1-z) + \left( \frac{4}{1-z} - 1 - z \right) \ln^2 z - 4 \frac{(1+z)^2}{1-z} \ln \frac{1-z}{\Delta} \left. \right] \tilde{\Sigma}(\hat{x}, \hat{y}, \hat{Q}^2) \\ & + \mathcal{O}(\text{const}), \end{aligned} \quad (9)$$

where  $P_\Theta^{(2)}(z)$  can be found in, e.g., [8, 9].

In contrast to the above contributions, the contributions from the regions *ii*) and *iii*) depend on the experimental determination of the kinematic variables and on the experimental selection of the events, as discussed below.

Within the required logarithmic accuracy it can be shown that the contribution from the semi-collinear region *iii*) factorizes as

$$\frac{1}{\hat{y}} \frac{d^3\sigma_{iii}^{\gamma\gamma}}{d\hat{x} d\hat{y} dz} = \frac{\alpha^2}{\pi^2} P(z, L_0) \int \frac{d^3k_2}{|\vec{k}_2|} \frac{\alpha^2(Q_h^2)}{Q_h^4} I^\gamma(zp, p', k_2), \quad (10)$$

with the precise form of the radiation kernel  $I^\gamma$  depending on the determination of the kinematic variables; for details we refer to [9, 10]. In the phase space integration over the hard photon, it is understood that the angular part of the  $k_2$ -integration is clearly restricted to the kinematic region *iii*), i.e., the full solid angle with the exception of the separately treated cones around the incoming and outgoing electron.

Finally we turn to the kinematic region *ii*). As discussed in [9], the contribution of this region to the observed cross section depends on the experimental event selection, i.e., on the method of measurement of the scattered particles. We shall focus on two scenarios. The first one is denoted as an *exclusive* (or bare) event selection, as only the scattered electron is measured; the hard photon that is emitted almost collinearly (i.e., within a small cone with opening angle  $2\vartheta'_0$  around the momentum of the outgoing electron) remains undetected or is not taken into account in the determination of the kinematic variables. The second case is a *calorimetric* event selection, when only the sum of the energies of the outgoing electron and

photon is actually measured if the photon momentum lies inside a small cone with opening angle  $2\vartheta'_0$  along the direction of the final electron.

For the exclusive event selection, when only the scattered electron is detected, we obtain

$$\frac{1}{\hat{y}} \frac{d^3\sigma_{ii,\text{excl}}^{\gamma\gamma}}{d\hat{x} d\hat{y} dz} = \frac{\alpha^2}{4\pi^2} P(z, L_0) \int_{\zeta_{\min}}^{\zeta_{\max}} \frac{d\zeta}{\zeta^2} \left[ \frac{1+\zeta^2}{1-\zeta} (\tilde{L} - 1) + (1-\zeta) \right] \tilde{\Sigma}_f(x_f, y_f, Q_f^2), \quad (11)$$

where  $\tilde{\Sigma}_f(x_f, y_f, Q_f^2)$  is an implicit function of  $\zeta$  via the relation between the “internal” kinematic variables  $x_f, y_f, Q_f^2$  and the “external” ones  $\hat{x}, \hat{y}, \hat{Q}^2$  (see refs. [8–10] for more details). Also, the large logarithm  $\tilde{L}$  generally depends on  $\zeta$ . The integration limits explicitly depend on the chosen determination of kinematic variables.

In the case of a calorimetric event selection, where only the sum of the energies of the outgoing electron and collinear photon is measured, the corresponding contribution reads

$$\frac{1}{\hat{y}} \frac{d^3\sigma_{ii,\text{cal}}^{\gamma\gamma}}{d\hat{x} d\hat{y} dz} = \frac{\alpha^2}{4\pi^2} P(z, L_0) \int_0^{\zeta_{\max}} d\zeta \left[ \frac{1+\zeta^2}{1-\zeta} (L'_0 - 1 + 2\ln\zeta) + (1-\zeta) \right] \tilde{\Sigma}(\hat{x}, \hat{y}, \hat{Q}^2), \quad (12)$$

where  $L'_0 = \ln(E'_e{}^2 \vartheta'_0{}^2 / m^2)$  is a large logarithm that now depends on the resolution parameter  $\vartheta'_0$  [8–10].

The total contribution from QED radiative corrections is finally found by adding up (6), (9), (10), and, depending on the chosen event selection, (11) or (12). The unphysical IR regularization parameter  $\Delta$  cancels in the sum, as it should.

It is important to note that the angle  $\vartheta'_0$  plays only the rôle of an intermediate regulator for the bare event selection and therefore drops out in the final result. In the calorimetric case there are no large logarithmic contributions from final state radiation as long as  $\vartheta'_0$  does not become too small. For more details see [9].

## 4 Numerical Results

In this section we shall present numerical results obtained for the leading and next-to-leading radiative corrections. As input we used

$$E_e = 27.5 \text{ GeV}, \quad E_p = 820 \text{ GeV}, \quad \vartheta_0 = 0.5 \text{ mrad}. \quad (13)$$

We chose the ALLM97 parameterization [13] as structure function with  $R = 0$ , no cuts were applied to the phase space of the second photon, and we assumed a calorimetric event selection. For the sake of simplicity we took a fixed representative angular resolution of  $\vartheta'_0 = 50$  mrad for the electromagnetic calorimeter to separate nearby hits by a scattered electron and a hard photon, which is close to realistic for the H1 detector at HERA. Also we disregard any effects due to the magnetic field bending the scattered charged electron away from a collinear photon.

Below we shall consider two methods that are used for the kinematic reconstruction of the radiative events and the determination of the kinematic variables: the electron method and the  $\Sigma$  method [14].

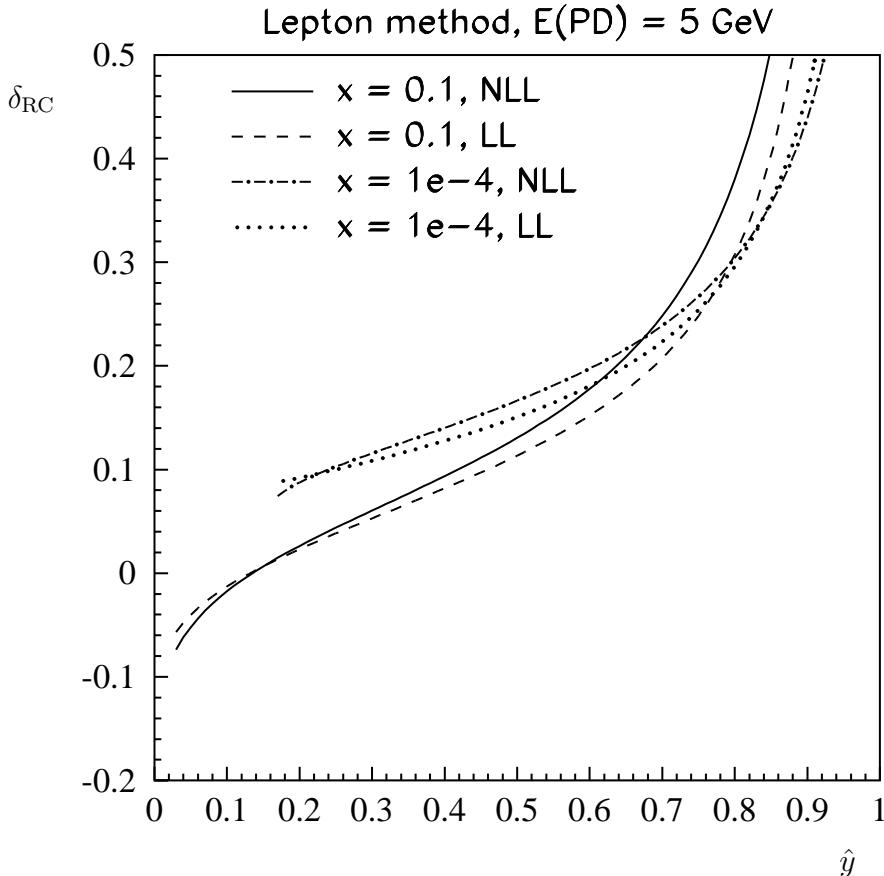


Figure 1: Radiative corrections  $\delta_{\text{RC}}$  (14) with leading and next-to-leading logarithmic accuracy at  $\hat{x} = 0.1$  and  $\hat{x} = 10^{-4}$  and a tagged photon energy of 5 GeV for the electron method. No cuts have been applied to the phase space of the second (semi-collinear) photon.

Figure 1 compares the radiative correction

$$\delta_{\text{RC}} = \frac{d^3\sigma}{d^3\sigma_{\text{Born}}} - 1 \quad (14)$$

calculated with leading and next-to-leading logarithmic accuracy for the electron method at  $\hat{x} = 0.1$  and  $\hat{x} = 10^{-4}$  and for a tagged energy of  $E_{\text{PD}} = 5$  GeV. Similar to the well-known QED corrections for DIS (see e.g., [15] and references cited therein), the corrections are large and positive for large  $\hat{y}$ , while they are large and negative for  $\hat{y} \rightarrow 0$  at large  $\hat{x}$ .

Figure 2 shows the corresponding corrections for the case of the determination of the kinematic variables using the  $\Sigma$  method [10]. In contrast to the electron method the corrections appear to be rather small, being only of the order of 5%, which can be easily traced back to the weak dependence of the kinematic variables on undetected initial state radiation. This leads to an almost complete cancelation of the leading logarithmic contributions to the corrections. On the other hand, the pure next-to-leading logarithmic parts of the corrections turn out to contribute significantly due to this suppression.

For this reason we should expect a relatively strong dependence of the corrections on the experimental selection of the events for the  $\Sigma$  method. This is illustrated in figure 3, that

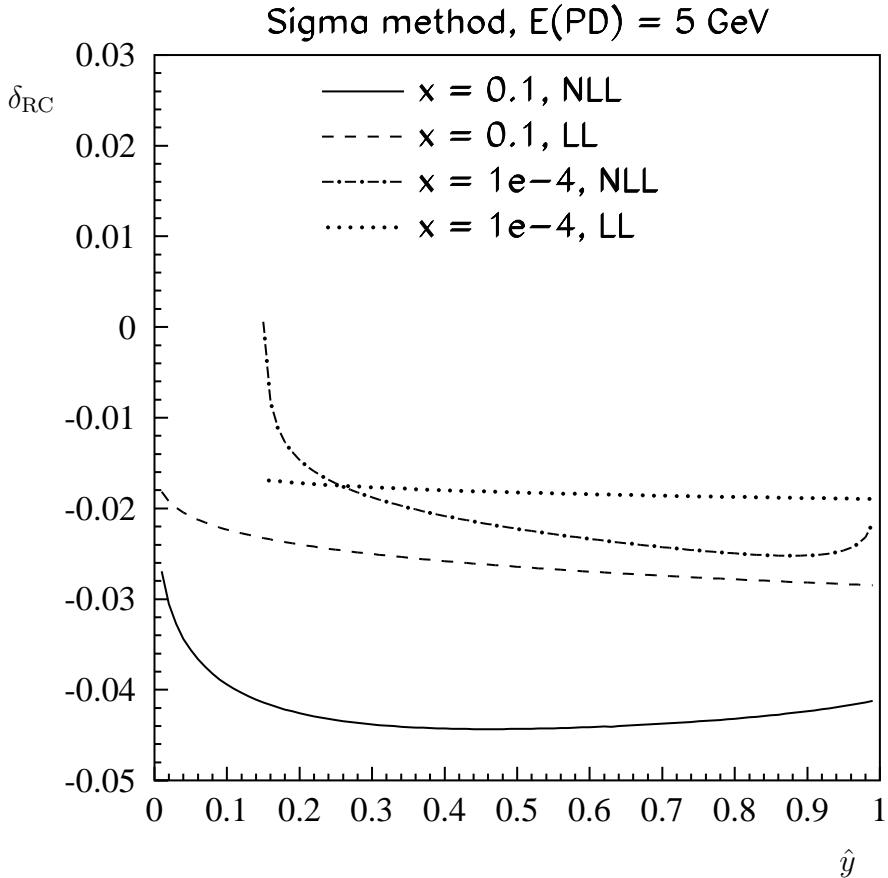


Figure 2: Radiative corrections at  $\hat{x} = 0.1$  and  $\hat{x} = 10^{-4}$  and a tagged photon energy of 5 GeV for the  $\Sigma$  method. As is figure 1, no cuts have been applied.

compares the QED corrections determined for the calorimetric event selection described above to a bare measurement of the scattered electron. Indeed we find a significant effect especially in the region of larger values of  $\hat{x}$ , where the corrections are dominated from soft photon emission.

To conclude, we have reported on the calculations of the QED corrections to DIS with a tagged photon. A semi-analytical program that incorporates these corrections is available on request from one of the authors (H.A.). Although no dedicated Monte Carlo event generator has been written yet, the implementation of the above results should be straightforward.

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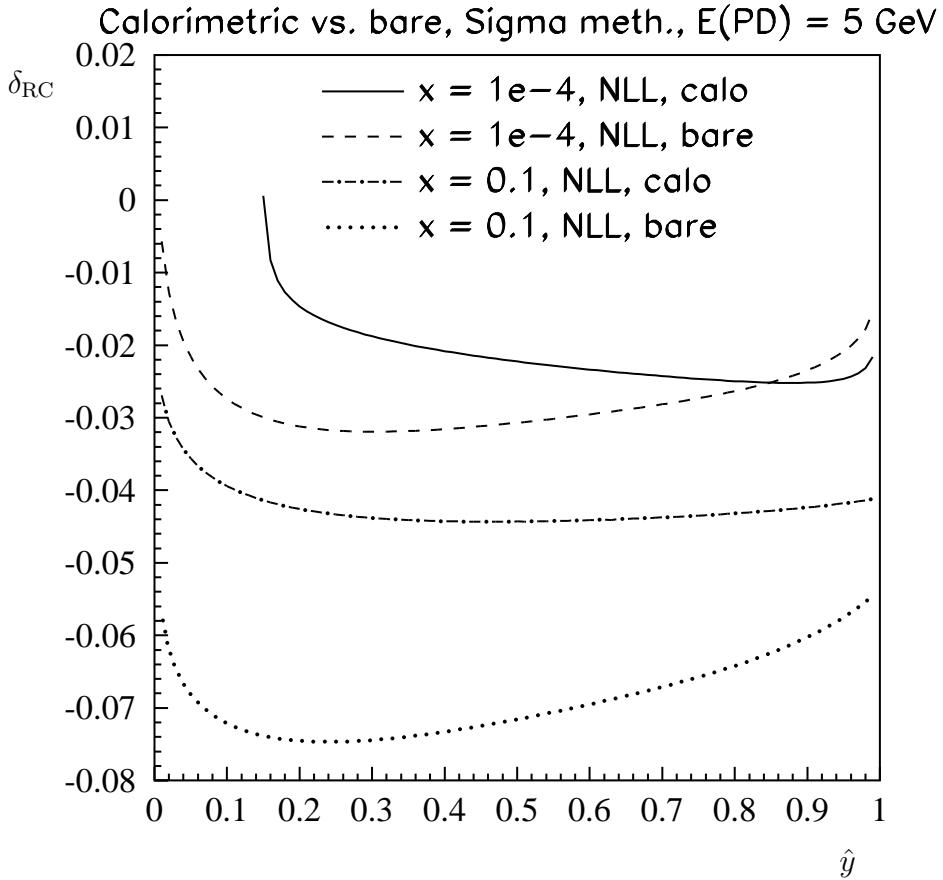


Figure 3: Comparison of the corrections for calorimetric (“calo”) vs. bare measurement of the scattered electron at  $\hat{x} = 10^{-4}$  and  $\hat{x} = 0.1$  for a tagged energy of 5 GeV for the  $\Sigma$  method.

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